

# Electric Charge Quantization in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ Models

Phung Van Dong<sup>1</sup> and Hoang Ngoc Long<sup>2</sup>

*Institute of Physics, VAST, P. O. Box 429, Bo Ho, Hanoi 10000, Vietnam*

## Abstract

Basing on the general photon eigenstate and the anomaly cancelation, we have naturally explained the electric charge quantization in two models based on the  $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$  gauge group, namely in the minimal model and in the model with right-handed neutrinos. In addition, we have shown that the electric charges of the proton and of the electron are opposite; and the same happens with the neutron and the neutrino. We argue that the electric charge quantization is not dependent on the classical constraints on generating mass to the fermions, but it is related closely with the generation number problem. In fact, both problems are properly solved as the direct consequences of the fermion content under the anomaly free conditions.

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## 1 Introduction

The  $SU(2)_L \otimes U(1)_Y$  symmetry of the standard model (SM) is a partial unification of the weak and electromagnetic interactions. It leaves many striking features of the physics of our world unexplained. Some of these are the quantization of electric charge (ECQ) and the generation number problem (GNP). The hydrogen atom is known to be electrically neutral to extraordinary accuracy. This implies that there is a relation between the charges of the quarks and of the electron. However, the electric charge operator in the SM owned with the form  $Q = T_3 + Y$ , while it can describe the observed charges, does not explain them. The problem is the  $Y$  generator. The values of  $T_3$  are quantized because of the non-Abelian nature of the  $SU(2)$  algebra. However, the values of  $Y$  are completely arbitrary [1, 2]. They are chosen to describe the discrete charges. However, as in the grand unified theory (GUT) [3], both  $T_3$  and  $Y$  are embedded into the  $SU(5)$  simple group, thus the values of  $Y$ , like those of  $T_3$  are constrained by the structure of the algebra, hence the ECQ has been derived. However, like the SM, the GUT also cannot explain the GNP; moreover, this simplest version of the GUT is fairly convincingly ruled out by the experiments on proton decay.

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<sup>1</sup>Email: pvdong@iop.vast.ac.vn

<sup>2</sup>Email: hnlong@iop.vast.ac.vn

A very interesting alternative to explain the origin of the generations comes from the cancelation of chiral anomalies [4]. In particular, the models based on the  $G_{331} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X$  gauge group, also called 3-3-1 models [5, 6, 7], arise as a possible solution to this puzzle, since some of such models require three generations in order to cancel completely chiral anomalies. In addition, in the literature on the ECQ in some 3-3-1 models, some solutions have been explored [8]; thus, it is hoped that the ECQ will be solved. However, the status is still opened with many problems which are not solved or not cleared, namely, the ECQ is obviously not dependent on the condition to generate mass for leptons and quarks. The usual theoretical grounds are completely unrestricted such as the definition of the fermion content and the electric charge operator, the relation between the ECQ and the GNP. They are still kept as the open questions!

In this paper, we will prove that the ECQ exists in the minimal 3-3-1 model and in the 3-3-1 model with right-handed (RH) neutrinos. We see that alternative to the GUT in which the problem is solved on the algebra structure of the simple group; here, it is a direct consequence from the usual fermion content in those models. We argue that the solution for the ECQ should be based on general laws, such as the conservation of the electric charge, the parity invariance of the electromagnetic interaction and the anomaly cancelation.

The rest of this paper is organized as follows: In Sec.2 a brief review of the 3-3-1 models is presented. It is emphasized that the photon eigenstate is dependent only on form of the electric charge operator from which the ECQ is derived. Next, in Sec.3, the fermion content, the electric charge operator and the classification of the models are represented. In Sec.4, from parity invariance of the electromagnetic vertices and anomaly cancelation, the ECQ is obtained. Our conclusions are summarized in the last section - Sec.5.

## 2 Some remarks on the gauge sector

To proceed further, in this section we review some essential consequences on the gauge sector of any 3-3-1 model which has been verified in [9]. Basing on these and two general properties of the electromagnetic interaction-the conservation of the electric charge and the parity invariance, we get equations for the usual  $U(1)_X$  charges. Then, the ECQ in the 3-3-1 models is derived.

Suppose that, under the  $G_{331}$  symmetry, there are a fermion triplet  $\mathbf{3} = (f_u, f_d, f_s)_L^T$  which is composed of a doublet  $(f_u, f_d)_L^T$  and a singlet  $f_{sL}$  of the  $SU(2)_L$  group of the SM, and an electric charge operator ( $e$ ) in this basic owning the form

$$Q = T_3 + \beta T_8 + X. \quad (2.1)$$

To break symmetry spontaneously, in general, three Higgs triplets are introduced

$$\chi \sim (1, 3, X_\chi), \eta \sim (1, 3, X_\eta), \rho \sim (1, 3, X_\rho), \quad (2.2)$$

which must acquire the vacuum expectation values (VEVs) as follows

$$\langle \chi \rangle^T = \left( 0, 0, \frac{v_s}{\sqrt{2}} \right),$$

$$\begin{aligned}\langle\eta\rangle^T &= \left(\frac{v_u}{\sqrt{2}}, 0, 0\right), \\ \langle\rho\rangle^T &= \left(0, \frac{v_d}{\sqrt{2}}, 0\right).\end{aligned}\tag{2.3}$$

The  $G_{331}$  group is decomposed into the gauge group of the SM by the Higgs triplet  $\chi$ . Next, the gauge group of the SM is decomposed into the  $SU(3)_C \otimes U(1)_Q$  by the two remaining Higgs triplets  $\eta, \rho$ . To keep the conservation of the electric charge, the operator  $Q$  must annihilate the vacuums:  $Q\langle\chi\rangle = 0$ ,  $Q\langle\rho\rangle = 0$  and  $Q\langle\eta\rangle = 0$ , then we get

$$\begin{aligned}X_\eta &= -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}, \\ X_\rho &= \frac{1}{2} - \frac{\beta}{2\sqrt{3}}, \\ X_\chi &= \frac{\beta}{\sqrt{3}},\end{aligned}\tag{2.4}$$

which are the fixing conditions for the  $U(1)_X$  charges of the Higgs scalars. They yield

$$X_\eta + X_\rho + X_\chi = 0.\tag{2.5}$$

The mass Lagrangian for the neutral gauge bosons is given by [9]

$$\mathcal{L}_{mass} = \frac{1}{2} V^T M^2 V,\tag{2.6}$$

where  $V^T = (W^3, W^8, B)$  and

$$M^2 = \frac{1}{4} g^2 \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix},\tag{2.7}$$

with

$$\begin{aligned}m_{11} &= v_u^2 + v_d^2, \\ m_{12} &= \frac{1}{\sqrt{3}} (v_u^2 - v_d^2), \\ m_{13} &= \frac{t}{\sqrt{6}} \left[ v_u^2 \left( -1 - \frac{\beta}{\sqrt{3}} \right) - v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right) \right], \\ m_{22} &= \frac{1}{3} (v_u^2 + v_d^2 + 4v_s^2), \\ m_{23} &= \frac{t}{3\sqrt{2}} \left[ v_u^2 \left( -1 - \frac{\beta}{\sqrt{3}} \right) + v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right) - v_s^2 \frac{4\beta}{\sqrt{3}} \right], \\ m_{33} &= \frac{t^2}{6} \left[ v_u^2 \left( -1 - \frac{\beta}{\sqrt{3}} \right)^2 + v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right)^2 + v_s^2 \left( \frac{2\beta}{\sqrt{3}} \right)^2 \right].\end{aligned}$$

Here  $t \equiv g_X/g$  with  $g_X, g$  are the gauge coupling constants of the  $U(1)_X$  and  $SU(3)_L$  groups, respectively. We have shown [9], for any 3-3-1 model (containing Higgs triplets,

antitriplets as well as sextets or any necessary Higgs scalar), the mass matrix of the neutral gauge bosons always has the above form. In addition, if additional Higgs scalars have non-zero VEVs, one just makes the following appropriate replaces

$$\begin{aligned} v_u^2 &\rightarrow v_u^2 + v_{u1}^2 + v_{u2}^2 + \dots, \\ v_d^2 &\rightarrow v_d^2 + v_{d1}^2 + v_{d2}^2 + \dots, \\ v_s^2 &\rightarrow v_s^2 + v_{s1}^2 + v_{s2}^2 + \dots, \end{aligned}$$

where  $v_{ui}, v_{dj}, v_{sk}$  are the VEVs of the neutral members in the additional Higgs, respectively. Thus, this is the general form of the mass matrix for the neutral gauge boson sector.

It can be checked that the matrix  $M^2$  has a *non-degenerate* zero eigenvalue. Therefore, the zero eigenvalue is identified with the photon mass,  $M_\gamma^2 = 0$ . The physical photon field  $A_\mu$  is directly defined from the equation  $M^2 A_\mu = 0$ :

$$A_\mu = \frac{t}{\sqrt{6 + (1 + \beta^2)t^2}} W_\mu^3 + \frac{\beta t}{\sqrt{6 + (1 + \beta^2)t^2}} W_\mu^8 + \frac{\sqrt{6}}{\sqrt{6 + (1 + \beta^2)t^2}} B_\mu. \quad (2.8)$$

Hence, for any 3-3-1 model, the photon eigenstate and mass are *independent* on the VEVs structure. These are a natural consequence of the  $U(1)_Q$  invariance - the conservation of the electric charge. Moreover, to be consistent with the QED based on the unbroken  $U(1)_Q$  gauge group, the photon field has to keep *the general properties of the electromagnetic interaction in the framework of the 3-3-1 model, such as the parity invariant nature* [10] (for more discussions, see [11]). These would help us to obtain some consequences related to quantities which are independent on VEVs structure such as the matching of gauge coupling constants [9]<sup>3</sup> and the ECQ.

Next, using (2.8) we write the coupling of the up member with the photon,  $\bar{f}_u f_u \gamma$  (also see [9]):

$$\begin{aligned} \mathcal{L}_{\bar{f}_u f_u \gamma}^{em} &= \bar{f}_{uL} i \gamma^\mu \left[ \frac{ig}{2} \frac{t}{\sqrt{6 + (1 + \beta^2)t^2}} + \frac{ig}{2\sqrt{3}} \frac{\beta t}{\sqrt{6 + (1 + \beta^2)t^2}} \right. \\ &\quad \left. + \frac{ig_X}{\sqrt{6}} X_3 \frac{\sqrt{6}}{\sqrt{6 + (1 + \beta^2)t^2}} \right] A_\mu f_{uL} \\ &\quad + \bar{f}_{uR} i \gamma^\mu \left[ \frac{ig_X}{\sqrt{6}} X_{f_u} \frac{\sqrt{6}}{\sqrt{6 + (1 + \beta^2)t^2}} \right] A_\mu f_{uR} \\ &= - \frac{(X_3 - X_\eta) g_X}{\sqrt{6 + (1 + \beta^2)t^2}} \bar{f}_{uL} \gamma^\mu f_{uL} A_\mu - \frac{X_{f_u} g_X}{\sqrt{6 + (1 + \beta^2)t^2}} \bar{f}_{uR} \gamma^\mu f_{uR} A_\mu, \end{aligned}$$

where  $X_3$  and  $X_{f_u}$  are the  $U(1)_X$  charges of the **3** triplet and of the  $f_{uR}$  singlet, respectively. Since the electromagnetic interaction is invariant under the parity transformation [10], then we get

$$X_{f_u} = X_3 - X_\eta. \quad (2.9)$$

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<sup>3</sup>As a result, the condition for matching of the gauge coupling constants in any 3-3-1 model is very natural as done in the SM.

Similarly for the vertices  $\bar{f}_d f_d \gamma$  and  $\bar{f}_s f_s \gamma$ , we get

$$X_{f_d} = X_3 - X_\rho, \quad (2.10)$$

$$X_{f_s} = X_3 - X_\chi, \quad (2.11)$$

where  $X_{f_d}$ ,  $X_{f_s}$  are the  $U(1)_X$  charges of the  $SU(3)_L$  singlets  $f_{dR}$  and  $f_{sR}$ , respectively. Note that  $f_{uR}$ ,  $f_{dR}$  and  $f_{sR}$  are the right-handed counterparts of the triplet  $(f_u, f_d, f_s)_L^T$ .

For a fermion antitriplet  $\mathbf{3}^* = (f'_d, -f'_u, f'_s)_L^T$  which is composed of an antidoublet  $(f'_d, -f'_u)_L^T$  and a singlet  $f'_s$  of the  $SU(2)_L$  group of the SM with  $f'_{uR}$ ,  $f'_{dR}$  and  $f'_{sR}$  are its right-handed counterparts, we also have

$$X_{f'_d} = X_{3^*} + X_\eta, \quad (2.12)$$

$$X_{f'_u} = X_{3^*} + X_\rho, \quad (2.13)$$

$$X_{f'_s} = X_{3^*} + X_\chi. \quad (2.14)$$

Here  $X_{3^*}$ ,  $X_{f'_u}$ ,  $X_{f'_d}$  and  $X_{f'_s}$  stand for the  $U(1)_X$  charges of the antitriplet and singlets  $f'_{uR}$ ,  $f'_{dR}$  and  $f'_{sR}$ , respectively.

As we will see, the lepton sector owns the triplets either  $\mathbf{3}_l = (\nu_l, l, \nu_l^C)_L^T$  or  $\mathbf{3}_l = (\nu_l, l, l^C)_L^T$ , where  $\nu_l^C = (\nu_{lR})^C$ ,  $l^C = (l_R)^C$  with  $C$  is the conjugate operator and  $l$  stands for the lepton  $e, \mu, \tau$ . Then, the two equations either (2.9) and (2.11) or (2.10) and (2.11) are replaced by equation one either

$$X_{3_l} = \frac{X_\eta + X_\chi}{2}, \quad (2.15)$$

or

$$X_{3_l} = \frac{X_\rho + X_\chi}{2}, \quad l = e, \mu, \tau, \quad (2.16)$$

respectively, which is directly obtained from the vertex  $\bar{\nu}_l \nu_l \gamma$  or  $\bar{l} l \gamma$ . It is worth to mention on significance of the equation (2.15) or (2.16) which is *the fixing condition for the  $U(1)_X$  charge of the lepton triplet as a natural consequence of the lepton content*. Further, with anomaly cancelation, the charges for all remaining chiral fermions are also fixed. Hence, these give the constraints on the hypercharge values  $Y = \beta T_8 + X$  from which the ECQ in the 3-3-1 models will explicitly be explained. Otherwise, if (2.15), (2.16) do not exist in some lepton content, there is not the ECQ unless add auxiliary conditions [8] such as on Majorana neutrino mass, non-RH neutrino singlets, etc. Thus, this means that (2.15) or (2.16) is *quantized condition*.

It is to be emphasized that if the ECQ in the GUT has been found by the mean of the algebra structure of the simple group, here for the 3-3-1 models, by their fermion content (the fermion structure under the anomaly free conditions).

### 3 Fermion content

In the framework of the 3-3-1 models, the essential basic concepts for building the models such as the fermion representations (reps), the electric charge operator, the anomaly cancelation and the fermion content will be explained. However, for our

purpose in studying the ECQ, it is necessary to note that the electric charges of the particles will be kept as parameters.

The SM is well done with the  $SU(2)_L$  doublets for the left-handed chiral spinors and the  $SU(2)_L$  singlets for the right-handed chiral spinors. Each generation of the SM consists of

$$\begin{aligned} \text{the doublets:} & \quad (\nu_l, l)_L^T, (u^\alpha, d^\alpha)_L^T \\ \text{and the singlets:} & \quad \nu_{lR}, l_R, u_R^\alpha, d_R^\alpha, \end{aligned} \quad (3.1)$$

where  $L, R$  stand for the left-handed and the right-handed counterparts, respectively,  $\alpha$  is the color index. Here  $l = e, \mu, \dots$ ;  $\nu_l = \nu_e, \nu_\mu, \dots$ ;  $u = u, c, \dots$  and  $d = d, s, \dots$  are the lepton and the quark particles in each generation, respectively.

Under the gauge symmetry  $G_{331}$ , the fermions transform like triplets **3**, antitriplets **3\*** or singlets **1** of the  $SU(3)_L$  group. Requiring the models at low energy to be fitly with the SM, the  $G_{331}$  symmetry must be spontaneously broken down that of the SM. Thus, the triplets or antitriplets are composed of the doublets **2** or antitriplets **2\*** and singlets **1** of the  $SU(2)_L$  group of the SM. The decomposition rule into the SM for the triplets yields

$$(\nu_l, l, S^l)_L^T = (\nu_l, l)_L^T \oplus S_L^l, \quad (3.2)$$

$$(u, d, S^q)_L^T = (u, d)_L^T \oplus S_L^q. \quad (3.3)$$

Similarly for the antitriplets

$$(l, -\nu_l, S'^l)_L^T = (l, -\nu_l)_L^T \oplus S_L'^l, \quad (3.4)$$

$$(d, -u, S'^q)_L^T = (d, -u)_L^T \oplus S_L'^q, \quad (3.5)$$

where  $S^l, S'^l, S^q, S'^q$  stand for the lepton and quark singlets, respectively. Note that if  $(f_u, f_d)_L^T$  is a doublet of the  $SU(2)_L$ , then  $(f_d, -f_u)_L^T$  is its antidoublet, they are equivalent and real reps.

Since the right-handed leptons in (3.1) are color singlets, they are put in the singlet  $S^l$  or  $S'^l$  by two ways [5, 6]:

$$(\nu_{lR})^C = S_L^l, \text{ or } S_L'^l \quad (3.6)$$

and

$$(l_R)^C = S_L^l, \text{ or } S_L'^l. \quad (3.7)$$

However, under the  $G_{331}$  and the Lorentz invariance, we cannot put the left-handed antiquarks in the bottom (singlet) of the triplets, so the existence of the exotic quarks is not able to avoid in all 3-3-1 models. In addition, the exotic leptons can be also in the singlets (in bottom of the lepton triplets or antitriplets), however, they are not considered here.

The fermion content under the  $G_{331}$  symmetry must be satisfied with the following criteria:

1. All singlets of the lepton triplets and antitriplets are either  $(\nu_{lR})^C$  or  $(l_R)^C$ .

2. Both  $(\nu_l, l, \nu_l^C)_L^T$ ,  $(l', -\nu_{l'}, \nu_{l'}^C)_L^T$  and  $(\nu_l, l, l^C)_L^T$ ,  $(l', -\nu_{l'}, l'^C)_L^T$  are not conjugate pairs, namely triplet and antitriplet.

Without loss of generality due to the second criterion, we can put the left-handed leptons in the triplets. Hence, on the first criterion there are two models: with  $(l_R)^C$  called the minimal 3-3-1 model [5], and also for  $(\nu_{lR})^C$  called the 3-3-1 model with RH neutrinos [6]. In this paper, they are called *usual* 3-3-1 models.

Due to the conservation and additive nature of the electric charge, the eco must be embedded in the neutral generators of the  $SU(3)_L \otimes U(1)_X$  group:

$$Q = \alpha T_3 + \beta T_8 + \gamma X. \quad (3.8)$$

Here the  $SU(3)_L$  charges  $T_3 = \lambda_3/2$ ,  $T_8 = \lambda_8/2$  with  $\lambda_3, \lambda_8$  are the two diagonal Gell-Mann matrices, and  $X$  is the  $U(1)_X$  charge. Without loss of generality, the  $\gamma$  coefficient can be normalized to 1 due to a scaling symmetry,  $g_X \rightarrow \gamma g_X$ ,  $X \rightarrow X/\gamma$ , where  $g_X$  is the  $U(1)_X$  coupling constant [12]. Finally, the two remaining coefficients  $\alpha$  and  $\beta$  get the same dimension of the electric charge. At the breaking point, the  $SU(3)_L$  group is embedded properly in the  $SU(2)_L$  group of the SM, therefore, the gauge boson  $W$  takes an electric charged value equal  $+\alpha$  or  $-\alpha$ . To see this, we should apply the eco (3.8) on a  $SU(3)_L$  triplet,  $\mathbf{3} = (f_u, f_d, f_s)_L^T$ ,

$$\begin{aligned} \frac{1}{2}\alpha + \frac{1}{2\sqrt{3}}\beta + \gamma X_{\mathbf{3}} &= q_{f_u}, \\ -\frac{1}{2}\alpha + \frac{1}{2\sqrt{3}}\beta + \gamma X_{\mathbf{3}} &= q_{f_d}, \\ -\frac{1}{\sqrt{3}}\beta + \gamma X_{\mathbf{3}} &= q_{f_s}, \end{aligned} \quad (3.9)$$

with  $q_{f_u}$ ,  $q_{f_d}$  and  $q_{f_s}$  are the electric charges of the members. Hence, the electric charge of  $W$  is  $q_{f_u} - q_{f_d} = \alpha$ . The normalization of the eco is undetermined, however we can always use the freedom in assigning the scale of the electric charge by putting the charged  $W$  in unit,  $\alpha = 1$  [12]. The eco is given by

$$Q = T_3 + \beta T_8 + X. \quad (3.10)$$

When the  $Y$  hypercharge is embedded into the  $SU(3)_L \otimes U(1)_X$  group, it is a linear combination of two terms,  $Y = \beta T_8 + X$ . The first term in the  $SU(3)_L$  is constrained by the structure of the algebra therefore quantized [1]; and, the second term in the  $U(1)_X$  with the values are kept as undetermined parameters. This differs the GUT from enlarging to the simple group, hence the ECQ is a direct consequence. However, as in the previous section, since all  $X$  charges are fixed, the present ECQ signs a different structure which refers to the particle reps.

Noting that the presence of the coefficient  $\beta$  signifies that the first term of the  $Y$  hypercharge is not properly normalized to be one of the  $SU(3)_L$  generators which have their scale fixed by the non-linear commutation relations [2],

$$[T_a, T_b] = if_{abc}T_c,$$

with

$$\text{Tr}[T_a T_b] = \delta_{ab}/2.$$

The value of  $\beta$  is obtained by comparing in the fundamental rep the values of  $T_8$  and the hypercharge values of the particles in some multiplet.

The action of the  $\text{eco}$  on an antitriplet  $\mathbf{3}^* = (f'_d, -f'_u, f'_s)^T$  is thanked to the usual rule

$$Q\mathbf{3} = q_3\mathbf{3}, \quad (3.11)$$

which yields

$$Q\mathbf{3}^* = q_3\mathbf{3}^* = -q_3^*\mathbf{3}^*. \quad (3.12)$$

Noting on the minus sign in the r.h.s of (3.12), we get  $X_{3^*} = -X_3 = -\text{Tr}Q$ .

Demanding for the fermion  $\text{SU}(3)_C$  reps to be vector-like and the color number  $N_C = 3$ , we get the non-trivial triangular anomaly cancelation conditions [13, 14] as follows

$$[\text{SU}(3)_C]^2 \otimes \text{U}(1)_X \quad : \quad 3X_q^L - \sum_{\text{singlet}} X_q^R = 0, \quad (3.13)$$

$$[\text{SU}(3)_L]^3 \quad : \quad \frac{1}{2}A_{\alpha\beta\gamma} = 0, \quad (3.14)$$

$$[\text{SU}(3)_L]^2 \otimes \text{U}(1)_X \quad : \quad \sum_{\text{family}} X_l^L + 3 \sum_{\text{family}} X_q^L = 0, \quad (3.15)$$

$$\begin{aligned} [\text{Grav}]^2 \otimes \text{U}(1)_X \quad : \quad & 3 \sum_{\text{family}} X_l^L + 9 \sum_{\text{family}} X_q^L \\ & - 3 \sum_{\text{family}} \sum_{\text{singlet}} X_q^R - \sum_{\text{family}} \sum_{\text{singlet}} X_l^R = 0, \end{aligned} \quad (3.16)$$

$$\begin{aligned} [\text{U}(1)_X]^3 \quad : \quad & 3 \sum_{\text{family}} (X_l^L)^3 + 9 \sum_{\text{family}} (X_q^L)^3 \\ & - 3 \sum_{\text{family}} \sum_{\text{singlet}} (X_q^R)^3 - \sum_{\text{family}} \sum_{\text{singlet}} (X_l^R)^3 = 0. \end{aligned} \quad (3.17)$$

Here  $X_l^L$ ,  $X_q^L$ ,  $X_l^R$  and  $X_q^R$  refer to the  $\text{U}(1)_X$  charges of the left-handed lepton, quark triplets or antitriplets and the right-handed lepton, quark singlets, respectively.

The cancelation of the  $[\text{SU}(3)_L]^3$  anomaly (3.14) demands for the number of fermion triplets to be the same as that of antitriplets. As mentioned above the  $\text{SU}(2)_L$  doublet and antidoublet are equivalent and real, hence, all the left-handed leptons and quarks are always ordered in the doublets. Moreover, using two conditions such as some known fermion generations are completely free from anomaly and the anomaly over all the quark and lepton generations must be canceled, we deduce that the number of the quark generations must be equal to that of the leptons. So, if  $N_f$  is the number of the fermion generations; thus, it is also the number of the lepton triplets as mentioned above. And,  $k$  is the number of the quark generations which are ordered in the triplets; then, there are the remaining  $N_f - k$  quark generations therefore in the antitriplets, satisfying

$$N_f + 3k = 3(N_f - k) \Rightarrow N_f = 3k. \quad (3.18)$$



Hence, the generation number  $N_f$  is a multiple of three. If further, one adds the condition of the QCD asymptotic freedom, which is valid only when the quark generation number is to be less than five, then it follows that  $N_f$  is equal to 3, and hence  $k = 1$ .

Therefore, the classification of the 3-3-1 models is given as follows,

1. The 3-3-1 model with RH neutrinos which the fermion reps are ordered by

$$\left(\nu_l, l, \nu_l^C\right)_L^T \sim (1, 3, X_l^L), l = e, \mu, \tau, \quad (3.19)$$

$$l_R \sim (1, 1, X_l^R), \quad (3.20)$$

$$(u_{3L}, d_{3L}, s_{3L}) \sim (3, 3, X_{q_3}^L), \quad (3.21)$$

$$(u_{iL}, d_{iL}, s_{iL}) \sim (3, 3^*, X_{q_i}^L), i = 1, 2, \quad (3.22)$$

$$u_{iR}, u_{3R} \sim (3, 1, X_{u_i}^R), (3, 1, X_{u_3}^R), \text{ also for } d \text{ and } s. \quad (3.23)$$

2. The minimal 3-3-1 model with the fermion reps read

$$\left(\nu_l, l, l^C\right)_L^T \sim (1, 3, X_l^L), l = e, \mu, \tau, \quad (3.24)$$

$$\nu_{lR} \sim (1, 1, X_{\nu_l}^R), \quad (3.25)$$

$$(u_{3L}, d_{3L}, s_{3L}) \sim (3, 3, X_{q_3}^L), \quad (3.26)$$

$$(u_{iL}, d_{iL}, s_{iL}) \sim (3, 3^*, X_{q_i}^L), i = 1, 2, \quad (3.27)$$

$$u_{iR}, u_{3R} \sim (3, 1, X_{u_i}^R), (3, 1, X_{u_3}^R), \text{ also for } d \text{ and } s. \quad (3.28)$$

Here, the (c,f,X) denotes the respective quantum numbers to the color, the flavor and the X-charge, and  $s_a$ ,  $a = 1, 2, 3$  are the added exotic quarks.

## 4 The ECQ

Now we turn on the ECQ in the 3-3-1 models. We first deal with the minimal version.

### 4.1 The ECQ in the minimal model

It is known that, the electromagnetic interaction is invariant under parity transformation. Using this property and anomaly cancelation we will get the needed ECQ. Let us deal with the lepton sector.

#### 4.1.1 The ECQ in the lepton sector

For the minimal model with the given lepton triplets, using Eq. (2.16), we get

$$\begin{aligned} X_l^L &= \frac{X_\rho + X_\chi}{2} \\ &= -\frac{X_\eta}{2}, \quad l = e, \mu, \tau. \end{aligned} \quad (4.1)$$

Applying Eq. (2.9) for the neutrinos, we have

$$X_{\nu_l}^R = X_l^L - X_\eta = -\frac{3}{2}X_\eta, \quad l = e, \mu, \tau. \quad (4.2)$$

Therefore, the application of the eco on the lepton triplets and the neutrino singlets yields the electric charges for the leptons as follows

$$q_{\nu_l} = \frac{3 + \sqrt{3}\beta}{4}, \quad (4.3)$$

$$q_l = \frac{-1 + \sqrt{3}\beta}{4}, \quad l = e, \mu, \tau. \quad (4.4)$$

So, with the help of the parity invariance of the electromagnetic vertices for all leptons, the ECQ of the lepton sector is derived. The electric charges of all leptons in the model are defined in terms of the  $\beta$ .

#### 4.1.2 The ECQ in the quark sector

Applying the equations (2.9), (2.10) and (2.11) for the quark triplets; and, (2.12), (2.13) and (2.14) for the quark antitriplets, we get

$$X_{u_3}^R = X_{q_3}^L - X_\eta, \quad X_{u_i}^R = X_{q_i}^L + X_\rho, \quad (4.5)$$

$$X_{d_3}^R = X_{q_3}^L - X_\rho, \quad X_{d_i}^R = X_{q_i}^L + X_\eta, \quad (4.6)$$

$$X_{s_3}^R = X_{q_3}^L - X_\chi, \quad X_{s_i}^R = X_{q_i}^L + X_\chi. \quad (4.7)$$

With these equations, we can write all Yukawa couplings to generate mass to all quarks

$$\begin{aligned} \mathcal{L}_Y &= h_{33}^s \bar{q}_3 s_3 \chi + h_{ii}^s \bar{q}_i s_i \chi^* \\ &+ h_{33}^u \bar{q}_3 u_3 \eta + h_{ii}^u \bar{q}_i u_i \rho^* \\ &+ h_{33}^d \bar{q}_3 d_3 \rho + h_{ii}^d \bar{q}_i d_i \eta^* + h.c.. \end{aligned} \quad (4.8)$$

Since the CKM matrix is non-diagonal, there are the flavor mixing terms in the Lagrangian (4.8). Therefore, the some terms in (4.8) must be changed as follows [12]

$$\begin{aligned} \bar{q}_3 u_3 \eta &\rightarrow \bar{q}_3 u_a \eta, \quad a = 1, 2, 3, \\ \bar{q}_3 d_3 \rho &\rightarrow \bar{q}_3 d_a \rho, \\ \bar{q}_i u_i \rho^* &\rightarrow \bar{q}_i u_a \rho^*, \quad i = 1, 2, \\ \bar{q}_i d_i \eta^* &\rightarrow \bar{q}_i d_a \eta^*. \end{aligned} \quad (4.9)$$

Under the  $U(1)_Q$  invariance, we have

$$\begin{aligned} X_{u_1}^R &= X_{u_2}^R = X_{u_3}^R \equiv X_u^R, \\ X_{d_1}^R &= X_{d_2}^R = X_{d_3}^R \equiv X_d^R. \end{aligned} \quad (4.10)$$

Thus, it is easy to get

$$\begin{aligned} X_{q_1}^L &= X_{q_2}^L \equiv X_q^L, \\ X_{s_1}^R &= X_{s_2}^R \equiv X_s^R. \end{aligned}$$

Using the anomaly cancelation (3.15), we have

$$2X_q^L + X_{q3}^L = \frac{1}{2}X_\eta. \quad (4.11)$$

Combination of (4.6) and (4.10) yields

$$X_q^L + X_\eta = X_{q3}^L - X_\rho. \quad (4.12)$$

From (4.11) and (4.12) it follows

$$X_q^L = \frac{1}{6}(X_\eta + 2X_\chi), \quad (4.13)$$

$$X_{q3}^L = \frac{1}{6}(X_\eta - 4X_\chi). \quad (4.14)$$

With the help of (4.13) and (4.14), we can express all charges for all right-handed quark counterparts in terms of  $X_\eta$  and  $X_\chi$ , namely

$$X_u^R = -\frac{1}{6}(5X_\eta + 4X_\chi), \quad (4.15)$$

$$X_d^R = \frac{1}{6}(7X_\eta - 4X_\chi), \quad (4.16)$$

$$X_s^R = \frac{1}{6}(X_\eta + 8X_\chi), \quad (4.17)$$

$$X_{s3}^R = \frac{1}{6}(X_\eta - 10X_\chi). \quad (4.18)$$

The equations (4.13)-(4.18) are the fixing conditions for the X-charges of the quark reps, therefore the charges of all fermion reps are indeed fixed.

Knowing the X-charges of the multiplets, we get the electric charges of their members as follows

$$q_u = \frac{5 - \sqrt{3}\beta}{12}, \quad u = u, \quad c, \quad t, \quad (4.19)$$

$$q_d = -\frac{7 + \sqrt{3}\beta}{12}, \quad d = d, \quad s, \quad b, \quad (4.20)$$

$$q_{s3} = -\frac{1 + 7\sqrt{3}\beta}{12}, \quad (4.21)$$

$$q_s = -\frac{1 - 5\sqrt{3}\beta}{12}, \quad s = s_1, \quad s_2. \quad (4.22)$$

In addition, it is easy to check that all the remaining anomaly cancelation conditions (3.13), (3.16) and (3.17) are satisfied. So, the ECQ of the quark sector is also given with the help of the anomaly cancelation.

We have the following remarks:

1. We can check the electric charge of the proton composed of three quarks  $uud$

$$\begin{aligned} q_p &= 2q_u + q_d \\ &= \frac{1 - \sqrt{3}\beta}{4}, \end{aligned} \quad (4.23)$$

which yields

$$q_p = -q_e. \quad (4.24)$$

2. The neutron is composed of three quarks  $ddu$ ; therefore, its electric charge is given by

$$\begin{aligned} q_n &= q_u + 2q_d \\ &= -\frac{3 + \sqrt{3}\beta}{4}, \end{aligned} \quad (4.25)$$

which yields the following interesting consequence

$$q_n = -q_p. \quad (4.26)$$

As mentioned above, the coefficient  $\beta$  should be fixed from the known hypercharge values in the SM [2]. Hence, for a lepton triplet, we have

$$Y(3_l) = \left(-\frac{1}{2}, -\frac{1}{2}, +1\right)^T = \beta T_8 + X_l^L. \quad (4.27)$$

From (4.1) and (4.27), it follows that  $\beta = -\sqrt{3}$ . Then the electric charges get the correct values as follows

$$q_{\nu_e} = 0, \quad e = e, \mu, \tau, \quad (4.28)$$

$$q_e = -1, \quad e = e, \mu, \tau, \quad (4.29)$$

$$q_u = +\frac{2}{3}, \quad u = u, c, t, \quad (4.30)$$

$$q_d = -\frac{1}{3}, \quad d = d, s, b, \quad (4.31)$$

$$q_{s_3} = +\frac{5}{3}, \quad (4.32)$$

$$q_s = -\frac{4}{3}, \quad s = s_1, s_2. \quad (4.33)$$

These relations have also been found in the literature [8], but they are based on the two principal conditions such as the classical constraints (to generate mass for the all fermions) and the anomaly cancelation.

## 4.2 The ECQ in the 3-3-1 model with RH neutrinos

For the 3-3-1 model with RH neutrinos, the  $\beta$  takes a value of  $-\frac{1}{\sqrt{3}}$ . Therefore, the exotic quarks get the electric charges different from those in the minimal model as follows

$$q_{s_3} = +\frac{2}{3}, \quad (4.34)$$

$$q_s = -\frac{1}{3}, \quad s = s_1, s_2. \quad (4.35)$$

This means that this model does not contain the exotic charges.

The electric charges of the usual leptons and quarks are the same as in the minimal model.

Thus, basing on the parity invariance of the electromagnetic interaction and the anomaly cancelation, we have shown that the usual 3-3-1 models contain in their framework the quantization of the electric charge.

## 5 Conclusions

Analyzing the photon eigenstate structure, we have shown that the general properties of the electromagnetic interaction such as the parity invariance is properly kept in the framework of the 3-3-1 models. This is a natural consequence of the conservation of the electric charge. As a result, the electric charge quantization in the usual 3-3-1 models has been derived.

Examining the fermion contents, we have found that the 3-3-1 models contain themselves two solutions such as the electric charge quantization and the generation number problem. Moreover, theoretically, the electric charges of the neutron and of the neutrino as well as of the proton and of the electron are opposite.

We pointed out that the electric charge quantization is independent on generating mass to the fermions. This is the main difference between our approach and that in [8], which was based on.

We have also shown that if in the GUT, the electric charge quantization results from the algebra structure, here in the 3-3-1 models based on the semi-simple group, it is a direct consequence of the fermion contents.

This conclusion adds one more nice feature to the 3-3-1 models.

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